

# The Mathematics of Investing



Tracking the performance of your investments can get confusing, due to the various ways of calculating returns. Whether you prefer to use a calculator or spreadsheet software, the following discussion will help you use and calculate common measurements of investment performance to truly judge your investment results.

## Determining Rate of Return

Probably the most basic calculation for investors is return on investment. Total return includes capital appreciation and income components, and assumes all income distributions are reinvested. If you automatically reinvest distributions such as interest or dividends, total return is calculated by taking the difference in an investment portfolio's ending and beginning balance, and dividing that difference by the beginning balance. In formula format, it would look like this:

$$\frac{\text{Total Return: (Calculator or Spreadsheet:)} \\ \text{(Ending Balance [EB] - Beginning Balance [BB])}}{\text{Beginning Balance}}$$

For example, Joe started with an investment of \$10,000. After five years, his portfolio's value increased to \$12,000. He can determine his portfolio's total return as follows:  $(\$12,000 - \$10,000) / \$10,000 = .20$ , or 20%. Therefore, Joe can say his \$10,000 has increased by 20%.

To annualize this total return, you'll need to calculate the compound annual return.

For example, Jane also originally invested \$10,000. However, it took her portfolio only two years to grow to \$12,000. If you measure the performances of both Joe's and Jane's portfolios by using the formula above, both increased by 20%. To take the difference in time into consideration, calculate the compound annualized rate of return (you will need a calculator that can raise to powers to calculate this).

**Compound Annualized Rate of Return =**  
**Calculator:  $[(EB / BB) ^ (1 / \# \text{ of years}) - 1]$**

**Spreadsheet:  $[(EB / BB) ^ (1 / \# \text{ of years})] - 1$**

Using this formula to calculate Joe's annual compound return, we take  $\$12,000 / \$10,000 = 1.2$ . Then, we raise 1.2 to the  $1/5$  (or 0.20) power, giving us 1.03714. Subtract out 1, and we have .03714, or 3.714%, which is Joe's annualized return. Jane's portfolio, on the other hand, performed much better, earning 9.54% on average every year. Of course, two different investments should not be judged solely on performance results for short periods of time or for different time periods. The risk of the portfolio must also be considered.

## 10% Plus 10% Doesn't Equal 20%

You might think that Jane's annualized return should have been 10%, and not 9.54%, since she invested her money for two years and  $10\% + 10\% = 20\%$ , which was her total rate of return. However, here's where the math can get tricky.

Let's just say that Jane's \$10,000 did grow 10% each year for two years. At the end of the first year, Jane would have accumulated \$11,000. In year one,  $\$10,000 \times (1 + .10) = \$11,000$ . In year two, if Jane's \$11,000 grows by another 10%, this gives us,  $\$11,000 \times (1 + .10) = \$12,100$ , which is more than the \$12,000 Jane actually accumulated. This \$100 discrepancy explains why Jane only earned a 9.54%, and not a 10%, compound annual return.

Similarly, the math doesn't intuitively make sense when you're losing money. If Jane's \$10,000 investment had lost 10% the first year, she would have \$9,000 left. In year two, if Jane's investment rebounds by exactly the same amount — 10% — Jane would not break even, as you might expect. In fact, a 10% increase in \$9,000

results in only \$9,900. Therefore, you need a greater percentage gain after a losing year in order to break even on your investment.

### The Rule of 72

If you need an approximation of how your nest egg might grow, you might want to use the Rule of 72. The Rule of 72 can reveal how long it could take your money to double at a particular rate of return. Use the following formula:

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#### Rule of 72

*72 / Annual Rate of Return = Number of years it will take for your money to double at a particular rate of return*

For example, Jane and Joe want to figure out how long it will take their \$10,000 investments to double to \$20,000. They use their compound annual rates of return (as figured previously) to estimate how many years it will take to double their money. Joe estimates it will take over 19 years ( $72 / 3.71\% = 19.4$  years). However, Jane's portfolio could grow to \$20,000 in less than eight years ( $72 / 9.54\% = 7.55$  years). It is important to note that the Rule of 72 does not guarantee investment results or function as a predictor of how your investment will perform. It is simply an approximation of the impact a targeted rate of return would have. Investments are subject to fluctuating returns, and there can never be a guarantee that any investment will double in value.

### Remember Taxes and Inflation

You should always take into consideration the effects of taxes and inflation when constructing an investment plan to meet your financial objectives. After all, even though Jane earned an average 9.54% on her investments every year, her "real" rate of return will be reduced by taxes and increases in the cost of living.

Depending on Jane's situation and income tax bracket, as much as 35% of her 9.54% compound annual return could be paid in federal taxes, leaving her with  $[9.54\% \times (1 - .35)]$ , or 6.20%.

Then, Jane must figure in the effects of inflation on her earnings. For example, assume inflation averaged 3% over the two years that Jane invested her \$10,000, and that she earned a 6.20% compound annual return after taxes, but before inflation. Now, Jane must adjust her after-tax return for the loss of purchasing power caused by inflation. To determine an inflation-adjusted rate of return, use the following formula:

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#### Inflation-Adjusted Return:

*(Calculator or Spreadsheet:)*

*$[(1 + \text{Rate of Return}) / (1 + \text{Inflation Rate}) - 1] \times 100$*

Jane's inflation-adjusted, after-tax rate of return is  $[(1.062) / (1.03) - 1] \times 100$ , or 3.11%. Keep in mind that we've assumed the highest federal income tax bracket (which does not apply to every investor); however, the example does show the impact that taxes and inflation can have on your return.

### Bond Yields

Bond investors generally receive periodic income from their investment. The amount of income paid to the holder of the bond is based on the bond's coupon rate. For instance, Jane buys a \$1,000 bond that pays a 7% coupon rate and therefore receives \$70 a year ( $\$1,000 \times .07$ ) for as long as she owns the bond. She can determine the income return (or yield) on this bond by taking the coupon dollar amount and dividing it by the purchase price of the bond, or  $\$70 / \$1,000 = 7.00\%$ . In this example, the yield and the coupon rate are the same because Jane purchased the bond at its original (or "face") value of \$1,000.

However, that yield can fluctuate depending on how much an investor pays for a bond. Let's say Jane's bond cost \$1,200. Its current yield is now only  $(\$70 / \$1,200)$ , or 5.83%.

Bond prices and yields may change over time with changes in interest rates. As the price of a bond increases, its yield decreases. Conversely, as

bond prices decrease, yields increase. If Jane's bond increases in value, her total return (income plus price appreciation) on the investment would be higher than the 7% coupon rate. However, as the yield on her bond changes, the dollar income she receives does not.

### Taxable-Equivalent Yield

Municipal bond investors generally receive income that is free from federal and in some cases state and local taxation. As a result, the stated yields on taxable bonds tend to be higher than yields on municipal bonds in order to compensate investors for their tax liability. When comparing bond yields, bond investors must use a taxable-equivalent yield to compare the rate of return on a tax-free municipal bond with that of a taxable bond.

The taxable-equivalent yield on a tax-free bond can be determined as follows:

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#### Taxable-Equivalent Yield :

*(Calculator or Spreadsheet:)*

*$\text{Tax-free yield} / (1 - \text{investor's marginal income tax rate})$*

For an investor in a 28% income tax bracket, the taxable equivalent yield for a municipal bond yielding 5% would be  $5\% / (1 - .28)$ , or 6.94%.

### No Substitute for Understanding

A financial planner can help you gauge your investments' performance, so you don't have to do the calculations yourself. But it is still your responsibility to understand what it all means. Without that knowledge, you could make potentially unfavorable financial decisions. With a fundamental understanding of the information presented above, you'll be better able to realistically judge your investments.

## Points to Remember

1. Total return is calculated assuming reinvestment of income distributions.
2. The annualized compound rate of return for any investment takes time into account.
3. The Rule of 72 gives you the number of years it will take your money to double at a particular return rate.
4. In pursuing your financial objective, you also need to take taxes and inflation into account.
5. Bond yields are based on a bond's coupon rate, but they can fluctuate depending upon the price paid for the bond. The yield can be either taxable or tax free.